An Introduction to Stochastic π-Calculus

Linguaggi e Modelli Computazionali L-S
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Outline

Quantitative Aspects in Process Algebras

- Process Algebras (PA) allow to model qualitative aspects of systems, i.e. behavioural descriptions
- Quantitative aspects are very important, particularly when dealing with concurrent or distributed systems
- Furthermore quantitative aspects allow to reason about performance

Probabilistic vs. Temporal Process Algebra

- An attempt to model quantitative aspects is to consider probability or time
- Probabilistic PA attach probabilities to branching point, ruling out nondeterminism
- Temporal PA associate a fixed duration to each transition to model execution time

Stochastic Process Algebra

- The approach followed by Stochastic PA is to add probabilistic distributions to prefixes
- The firing of a prefix occurs after a delay of Δt defined by the respective probabilistic distribution
- Such delay can model the actual duration of the action associated with the prefix

Exponential Distribution

- Most PAs in the literature are defined according to exponential distribution $P = 1 - e^{-rt}$, where the mean value is $1/r$
- Exponential distributions are completely characterised by a single parameter r
- Exponential distributions enjoy the memoryless property, that is the duration of a transition is independent from the history of transitions

Action Duration

- In stochastic PA a prefix is defined by a couple (a,r) where a is the action and r is the *activity rate*
- The activity rate denote the duration of an action according to the exponential distribution
- When r is specified the activity is termed *active* otherwise it is *passive*

Stochastic PA and Markov Chains

- Exponential Distribution allow to derive from a PA specification a continuous time Markov Chain (CTMC)
- A CTMC is a memoryless stochastic process, i.e. a collection of time dependent random variables following an exponential distribution [Brinksma and Hermanns, 2001]
- Markov Chains are commonly used in numerical techniques to obtain performance measures
- Particularly interesting for the performance evaluation task is the stochastic process algebra PEPA

More on Markov Chains

- A CTMC can be represented by a labelled transition system where nodes represent the states and transitions are labelled with rates [Brinksma and Hermanns, 2001]
- The probability for a specific transition to happen is given by the ratio between the rate and the sum of the exit rates

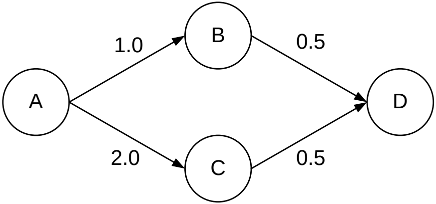
$$P_j = \frac{r_j}{\sum_i r_i}$$

- Given the selected transition, the duration of the actions is distributed according to the sum of rates

$$P = 1 - e^{-\sum_i r_i t}$$

CTMC Example

- Consider the CTMC in the Figure below: from the initial state A the probability to move to B is $P(A \rightarrow B) = 1/(1+2) = 0.\overline{3}$, while the probability to move to C is $P(A \rightarrow C) = 2/(1+2) = 0.\overline{6}$
- The duration of both transitions is distributed exponentially according to the sum of exit rates $r_t = 1.0 + 2.0 = 3.0$



Outline

π-Calculus Syntax

- A process in π-calculus is defined by Milner [Milner et al., 1992] [Milner, 1999] accordingly to the following syntax

$$P ::= 0 \mid P_1 + P_2 \mid \bar{y}x.P \mid y(x).P \mid \tau.P \mid P_1|P_2 \mid (\nu x)P \mid [x = y]P \mid !P$$

- 0 is the empty process, and it's called *inaction*, often omitted
- the silent prefix τ means that a silent action is performed
- the replication $!P$ means that you can have as many copies – but a finite number – as you wish, i.e. $P|P|P|..$

π-Calculus Syntax

- summation $P_1 + P_2$ means that the process can perform P_1 or P_2
- the prefix $\bar{y}x$ is a sort of output port, so $\bar{y}x.P$ means send x across y channel and then behave like P
- the prefix $y(x)$ is a sort of input port, so $y(x).P$ means receive a value across y channel, name it x and then behave like P
- the composition $P_1|P_2$ means that the two processes are executed in parallel
- the restriction $(\nu x)P$ means that the process behaves like P except for the fact that any action across x channel is prohibited
- $[x = y]P$ means that the process behaves like P if y matches x, otherwise 0

Stochastic π-Calculus Syntax

- The following description of the stochastic π-Calculus is based on the proposal in [Priami, 1995]
- Prefixes are annotated by the activity rate, i.e. prefixes become a pair (a,r) where a is the action and r is the activity rate
- The time to complete the activity Δt is drawn according to the exponential distribution defined by r

Race Condition

- In summation the first activity that completes is executed while the others are discarded
- All the activities enabled attempt to proceed, although only the fastest one is executed: this criteria is called *race condition*
- Each time the fastest activity may differ since execution time is a random variable, hence it involves a probabilistic choice

Probabilistic Choice

- The probability for an action to be executed is given by the ratio of its rate and the *exit rate*, i.e. the sum of the rates enabled
- For example in (a,2)+(b,6) a has a probability of $2/(2+6) = 0.25$ of being executed
- In general, the probability for an action i of being chosen among the j enabled actions is

$$p_i = \frac{r_i}{\sum_{j=1}^{n} r_j}, \quad 1 \leq i \leq n \ . \tag{1}$$

Growing Interest

- Interest in Stochastic π-Calculus has grown considerably in the last decade
- In particular it has been exploited for
  - modelling concurrent and distributed systems
  - evaluate performance of distributed systems
  - simulate and analyse systems, especially with applications in chemistry and biology

Outline

SPiM

- SPiM [Phillips and Cardelli, 2004][Phillips, 2007b] is a simulator for stochastic π-Calculus specifications, initially developed for investigating biological systems
- The correctness of the machine has been formally proven with respect to the calculus
- The machine defines a variant of $\pi - \mathit{calculus}$
- Notice also that for programming convenience SPiM language use a different syntax from standard π-Calculus: for details please refer to the SPiM Language Manual [Phillips, 2007a]

Stochastic Selection Algorithm

- The stochastic selection algorithm is based on [Gillespie, 1977] and exploits the notion of channel activity, which is defined by

$$Act_x(A) = (In_x(A) \times Out_x(A)) - Mix_x(A) \qquad (2)$$

- where $In_x(A)$ and $Out_x(A)$ are the number of unguarded inputs and outputs on channel x in A
- and $MiX_x(A)$ is the sum of $In_x(\Sigma_i) \times Out_x(\Sigma_i)$ for each Σ_i in A

Stochastic Selection Algorithm

- For all $x \epsilon fn(A)$ calculate $a_x = Act_x(A) \times rate(x)$
- Store non-zero values of a_x in a list (x_μ, a_μ) where $\mu \epsilon 1..M$
- Calculate $a_0 = \sum_{\nu=0}^{M} a_\nu$
- Generate two random numbers $n_1, n_2 \epsilon [0, 1]$ and calculate τ, μ such that:

$$\tau = (1/a_0) ln(1/n_1)$$

$$\sum_{\nu=1}^{\mu-1} a_\nu < n_2 a_0 \leq \sum_{\nu=1}^{\mu} a_\nu$$

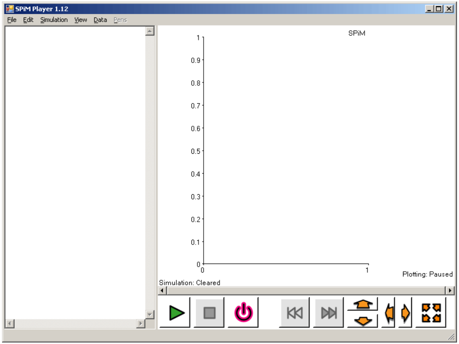
- $Next(A) = x_\mu$ and $Delay(A) = \tau$

SPiM Command Syntax

- In order to use SPiM you have to switch to shell mode: the command syntax is *spim filename.spi*
- As a convention SPiM specifications have the extension *.spi*
- By default, the results are written into a file having the same name of the specification appended with the extension *.csv*, i.e. comma separated values: for example, in the previous example, the output file would be *filename.spi.csv*
- Furthermore SPiM allows to export the specification to a graphical notation: the file has *.dot* extension and can be read by GraphViz [GraphViz, 2007]

SPiM Player

- As an alternative it is possible to use the SPiM player, a Graphical User Interface to SPiM command
- Since it has plotting capabilities it allows previewing the system dynamics without switching to another software



NaCl Example

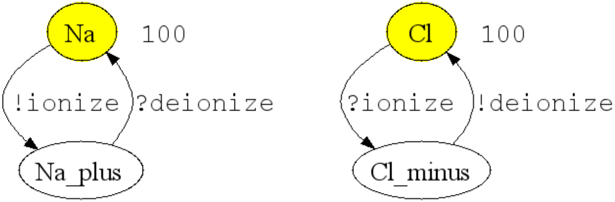


Figure: The representation of the NaCl specification using GraphViz [GraphViz, 2007].

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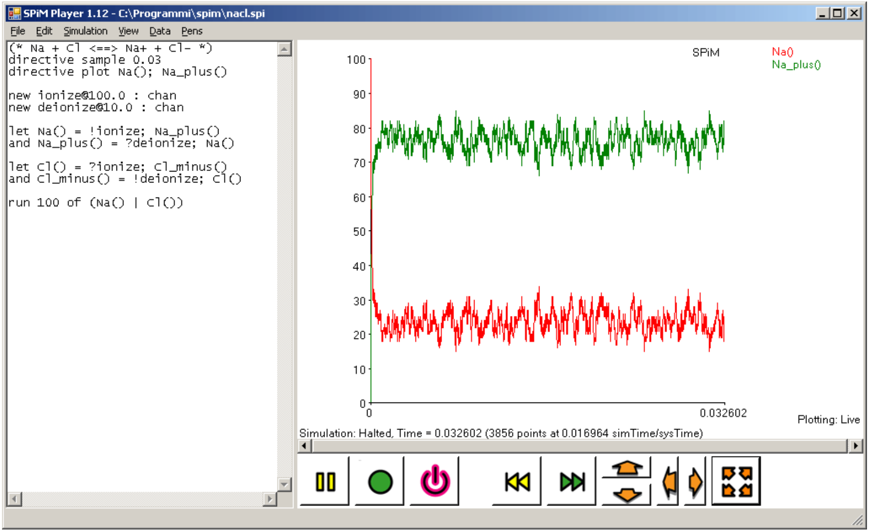

Dissecting the NaCl Specification

Directives

- directive sample 0.03
- directive plot Na(); Naplus()

Channel Declaration

- new ionize@100.0 : chan
- new deionize@10.0 : chan

Processes Definition

- let Na() = !ionize; Naplus() and Naplus() = ?deionize; Na()
- let Cl() = ?ionize; Clminus() and Clminus() = !deionize; Cl()

Run command

- run 100 of (Na() — Cl())

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Summing up

- Stochastic π-Calculus labels π-Calculus prefixes with rates: rates completely characterise exponential distributions
- This allows the definition of a stochastic selection algorithm and the mapping with Markov Chains
- The modelling bricks in Stochastic π-Calculus are Processes and Channels
- SPiM implements a variant of the Stochastic π-Calculus and allows to run simulations directly from the specifications for quantitative analysis

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