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# On pi-Calculus and its Applications

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# Outline

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## I. Pi-calculus

- Definition & syntax
- Extensions
- Tools

## II. On the applicability of pi-calculus to self-org. Systems

- Definition of self-org. system & emergent property
  - Pros and Cons
  - The role of Spi in our research
  - Our case study
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# Seven **Myths** of Formal Methods

## Hall 1990

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- 1) Formal methods can guarantee perfect software and eliminate the need for testing
- 2) Formal methods are all about proving programs correct
- 3) Formal methods are only useful in safety-critical systems
- 4) Formal methods require highly trained mathematicians
- 5) Formal methods increase development costs
- 6) Formal methods are unacceptable to users
- 7) Formal methods are not used on real large-scale systems

# Informal Definition

## Milner 1992

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Pi-calculus is a calculus for **communicating systems** in which one can naturally express processes which have changing structure.

Not only may the component agents of a system be arbitrarily linked, but a communication between neighbours may carry information which changes that linkage.

# About Milner's definition of Agent

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- In pi-calculus an agent is actually represented as a process
- Milner considers only agents with a finite behavior
- Milner gives no further details about agents, and usually the term agent is even omitted
- It also gives no details about the environment which has become one of the most important issues in MAS
- Hence in pi-calculus you can describe systems that can be conceived in terms of processes

# History

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- Pi-calculus is an extension of Milner's process algebra called **CCS** – Calculus for Communicating Systems
- It adds the feature of **process mobility**, i.e. processes having changing control structure, exploiting the results of Engberg and Nielsen
- CCS has been successfully applied for modelling several electronics and software systems

# Syntax

## Milner 1992 & 1993

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$$P ::= 0 \mid P_1 + P_2 \mid \bar{y}x.P \mid y(x).P \mid \tau.P \mid P_1|P_2 \mid (\nu x)P \mid [x = y]P \mid !P$$

- $0$  is the empty process, and it's called *inaction*: since it is very common it is often omitted;
- summation  $P_1 + P_2$  means that the process can perform either  $P_1$  or  $P_2$ ;
- the prefix  $\bar{y}x$  is a sort of output port, so  $\bar{y}x.P$  means send  $x$  across  $y$  channel and then behave like  $P$ ;
- the prefix  $y(x)$  is a sort of input port, so  $y(x).P$  means receive a value across  $y$  channel, name it  $x$  and then behave like  $P$ ;
- the silent prefix  $\tau$  means that a silent action is performed;
- the composition  $P_1|P_2$  means that the two processes are executed in parallel
- the restriction  $(\nu x)P$  means that the process behaves like  $P$  except for the fact that any action across  $x$  channel is prohibited;
- $[x = y]P$  means that the process behaves like  $P$  if  $y$  matches  $x$ , otherwise  $0$ ;
- the replication  $!P$  means that you can have as many copies – but a finite number – as you wish, i.e.  $P|P|P|\dots$

# Reduction Rules

## Milner 1992 / 1993

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$$COMM : (\dots + x(y).P) \mid (\dots + \bar{x}z.Q) \rightarrow P\{z/y\} \mid Q$$

$$PAR : \frac{P \rightarrow P'}{P \mid Q \rightarrow P' \mid Q}$$

$$RES : \frac{P \rightarrow P'}{(\nu x)P \rightarrow (\nu x)P'}$$

$$STRUCT : \frac{Q \equiv P \quad P \rightarrow P' \quad P' \equiv Q'}{Q \rightarrow Q'}$$



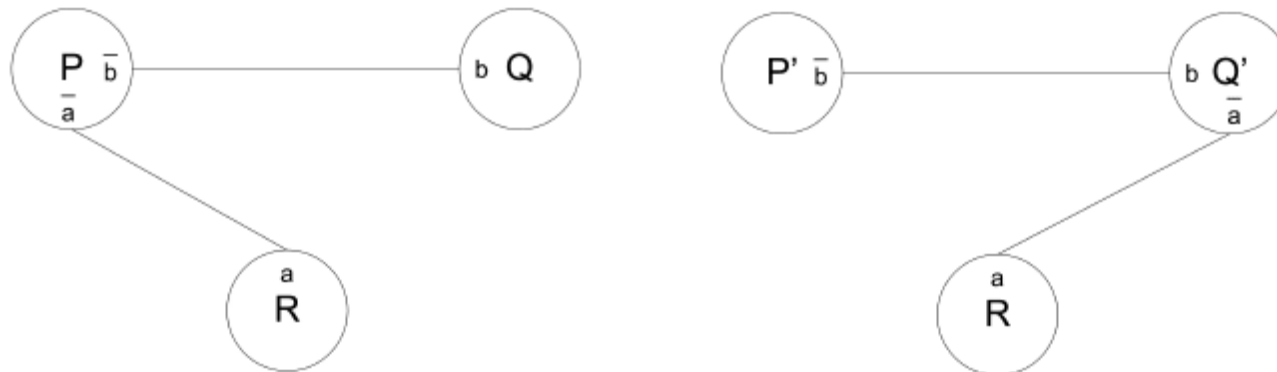
# Name Passing Example

## Milner 1992

- 3 processes P,Q,R: P and R share channel a, P and Q share b
- P wants to send the value 42 to R
- P wants to delegate that task to Q

$$(\nu a)(\nu b)(\bar{b}a.\bar{b}42.P' \mid b(x).b(y).\bar{x}y.0 \mid a(z).R')$$

$$(\nu a)(\nu b)(P' \mid \bar{x}y.0 \mid a(z).R')$$



# What can I do with Pi?

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- Investigate fundamental questions in concurrency & communication
- Formal proofs for SMALL systems
- Write programs in a “functional style” using tools like Pict ... but it's better suited as a framework for higher-level languages
- It's a good **modeling language**: e.g. network and security protocols have been successfully modeled in pi-calculus
- It can be used also for simulations... but it lacks one important feature: **stochasticity**

# Stochastic Extension to Pi

## Priami 1995

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- Choices in Pi are nondeterministic
- Priami introduced an extension to Pi in order to make choices probabilistic
- Each channel is associated to an *activity rate*  $r$
- The usage of a resource is a random variable with exponential distribution defined by  $r$
- Given  $P.(P_1+P_2+\dots+P_n)$  the probability of the transition between  $P$  and  $P_i$  is

$$p_i = \frac{r_i}{\sum_{j=1..n} r_j}, \quad 1 \leq i \leq n$$

# ...and now?

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- Once we have modeled our system with stochastic pi-calculus maybe we want to **simulate** its working for
  - Feasibility of the whole system
  - Assessment of global properties
  - Performance analysis
  - ...
- For that purpose we need an interpreter of Spi specifications

# Stochastic Pi Machine (SPiM)

## Phillips

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- The Stochastic Pi Machine (SPiM) is a simulator for the stochastic pi-calculus that can be used to simulate models of **Biological systems**.
- The machine has been formally specified, and the specification has been proved correct with respect to the calculus.
- It is a quite fast and optimized simulator
- The syntax is quite different from the one of pi-calculus and have been added more features

# SpiM: KNa2Cl example

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```
(* K + Na + 2Cl <==> K+ + Na+ + 2Cl- *)
directive sample 0.03
directive plot Cl(); Na_plus(); K_plus()

new ionize1@100.0:chan
new deionize1@10.0:chan
new ionize2@30.0:chan
new deionize2@20.0:chan

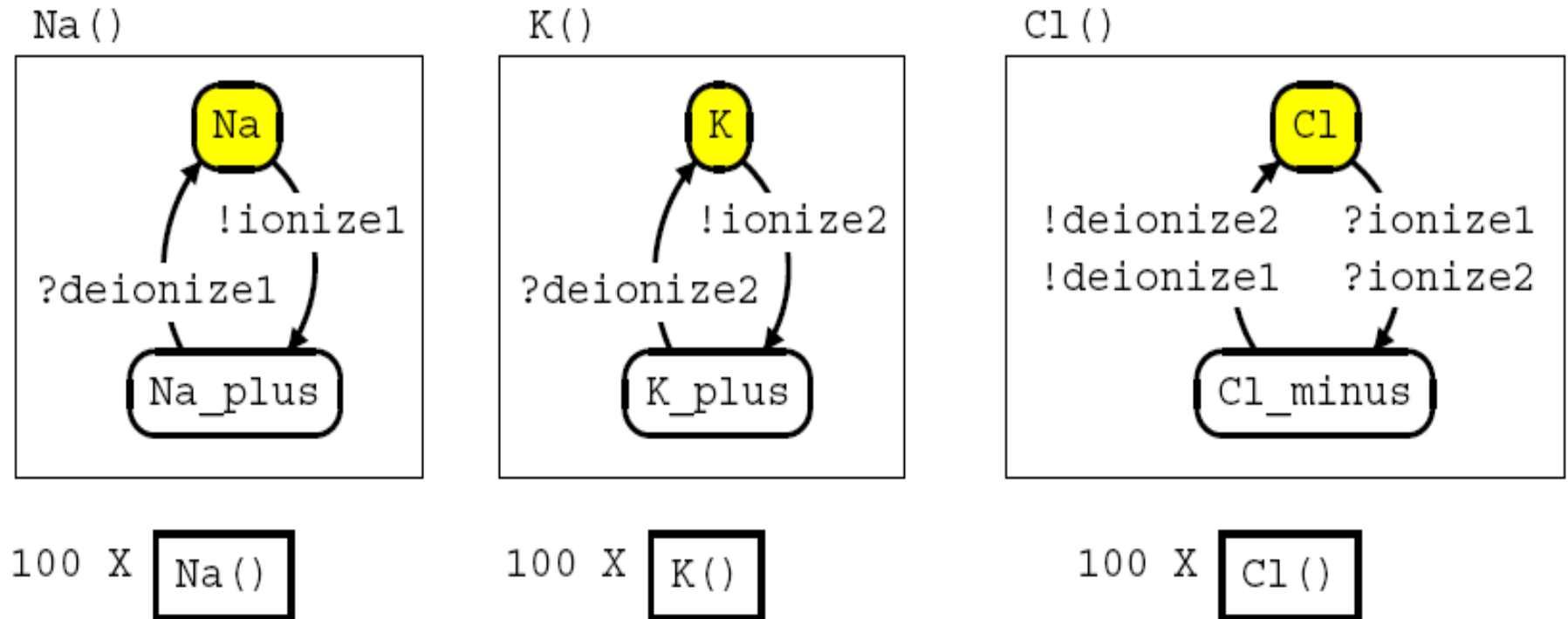
let Na() = !ionize1; Na_plus()
and Na_plus() = ?deionize1; Na()

let K() = !ionize2; K_plus()
and K_plus() = ?deionize2; K()

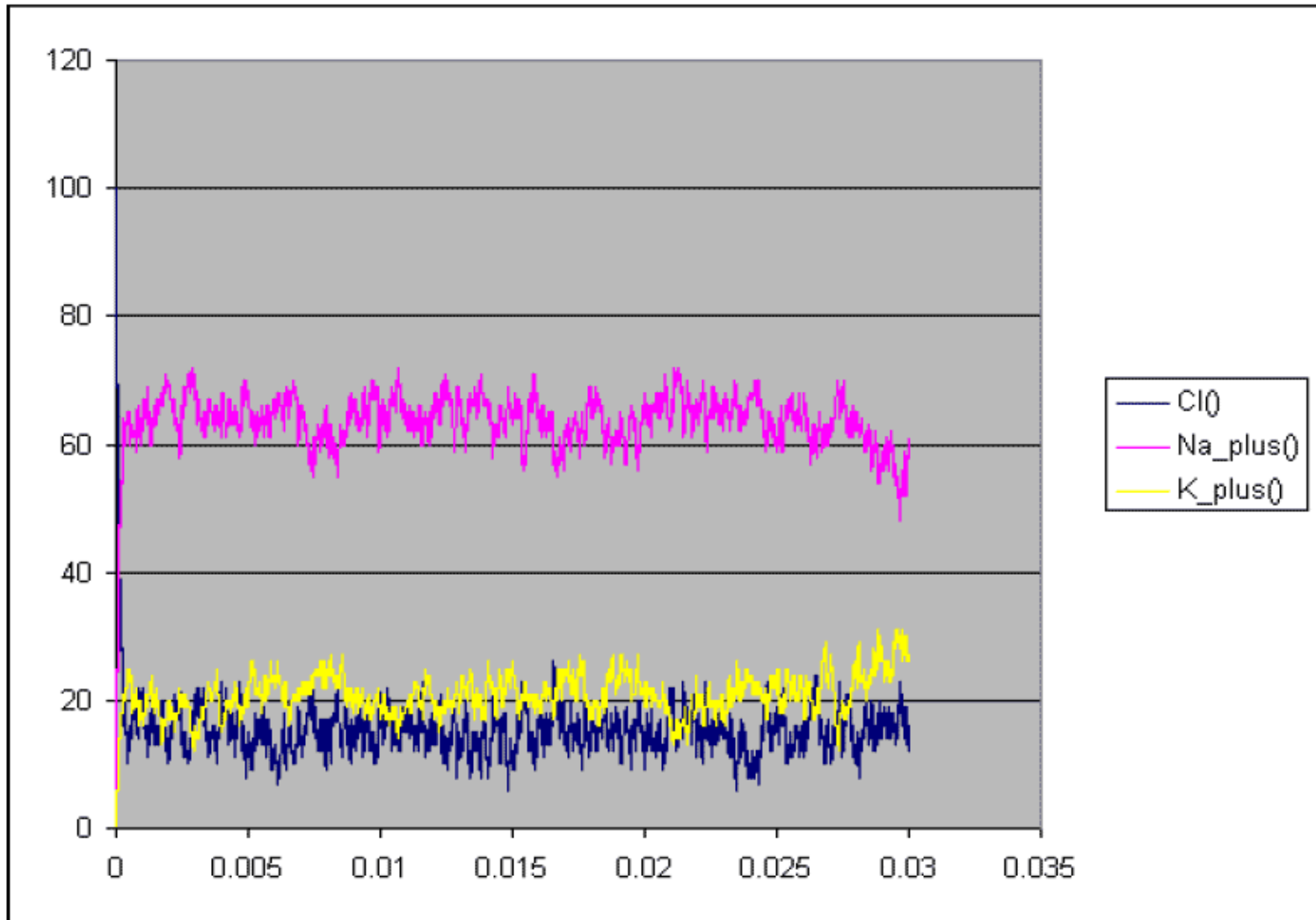
let Cl() =
  do ?ionize1; Cl_minus()
  or ?ionize2; Cl_minus()
and Cl_minus() =
  do !deionize1; Cl()
  or !deionize2; Cl()

run 100 of (Na() | Cl() | K())
```

# SpiM: KNa2Cl example



# SpiM: KNa2Cl example





# Comments

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- It is very simple to model systems similar to  $\text{KNa2Cl}$
- The mapping is quite easy when the essence is about signaling for inhibition or catalysis
- From a scientific it is very interesting to have such tool, but how it can usefully applied to MAS?

# Definition of Self-Organization

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A system is said to be self-organizing if it's able to re-organize itself upon environment changes autonomously via local interactions between its parts.

# Definition of Emergent Property

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A system property is said to be emergent if it cannot be described using the same - or equivalent - ontology used to describe the system itself.

# On the applicability of Spi-calculus to self-org. systems

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- Spi-calculus is not directly/strictly related to SOS
- Why we started considering Spi for SOS?
  - it is well suited for large scale systems with weak heterogeneity
  - it is a modeling language for communication / interaction
  - full compositionality
  - it has already been applied to biology/chemistry research, e.g. see Cardelli's works
  - it features stochasticity

# Limitations

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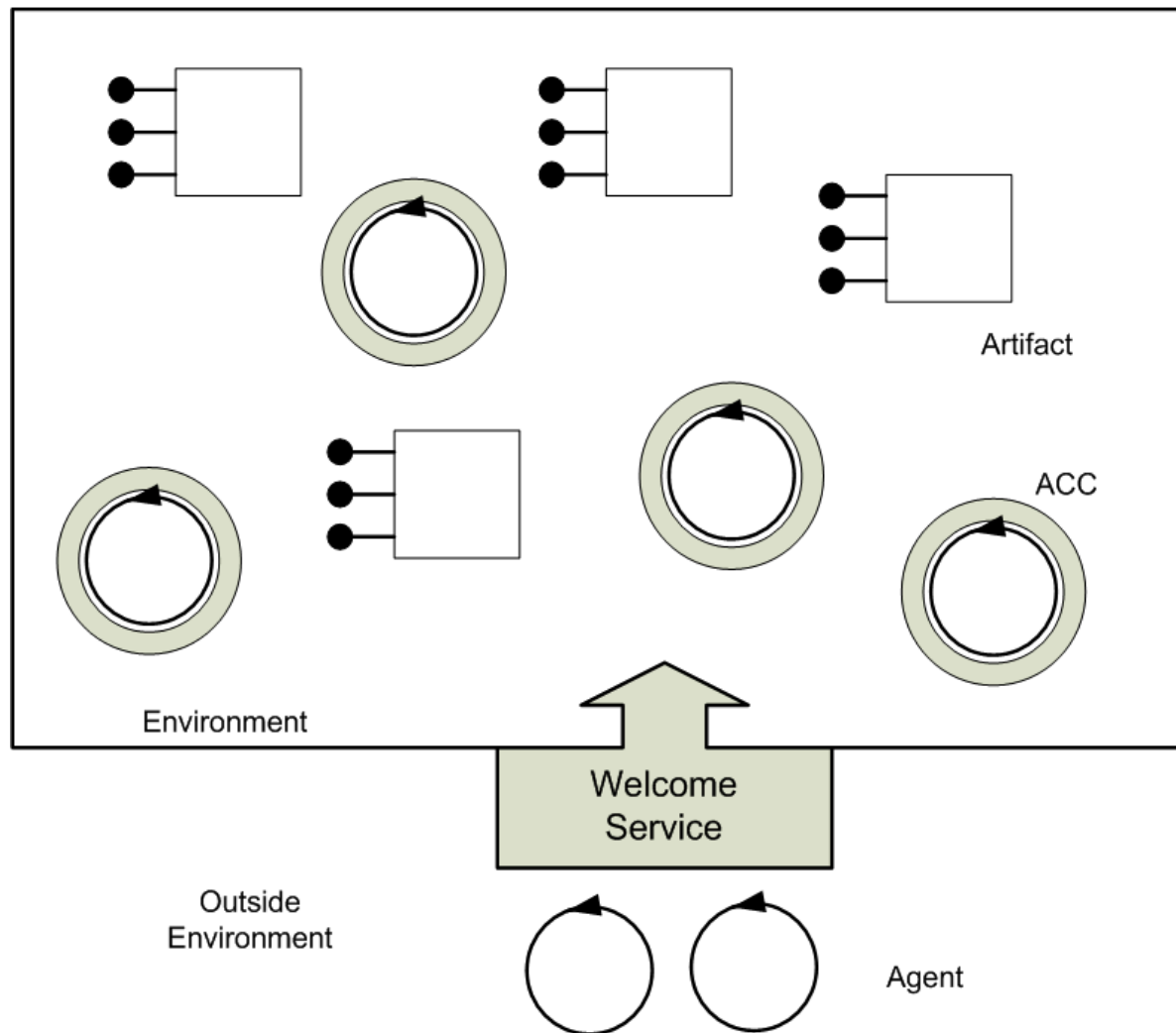
- Since it is quite young there are many issues that should be tackled
- e.g. it does not take into account *explicitly* locality issues: this might be an interesting point of extension
- Feasible models of complex heterogeneous systems can be very abstract / high-level

# Role of Spi in our research

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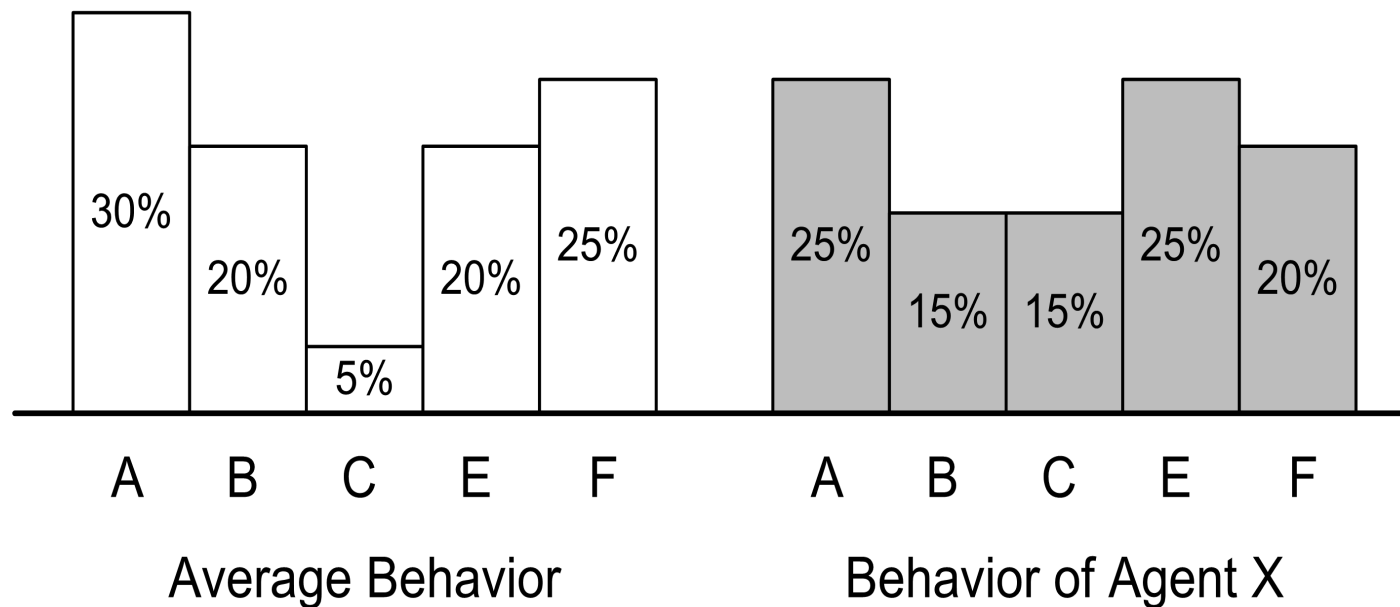
- 1) Feasibility study at early stages of analysis
- 2) “Observe” global system properties actually happening & detect emergent properties
- 3) Coarse tuning system parameters
- 4) Performance evaluation, e.g. PEPA

# Our case study



# Our case study

- We can detect abnormal behaving agents through analysis of the distribution of their actions
- If an agent X is behaving “differently” from the average – especially for critical actions – we may decide to further inspect him or deny access to resources





# Our case study

